

To find the eigenvalues, we solve

$$\mathbf{A} - \lambda I = \begin{pmatrix} a - \lambda & b \\ c & d - \lambda \end{pmatrix}.$$

Taking the determinant yields

$$\det(\mathbf{A} - \lambda I) = (a - \lambda)^2 - b^2 = \lambda^2 - 2a\lambda + (a^2 - b^2).$$

Setting this expression equal to zero gives

$$\lambda^2 - 2a\lambda + (a^2 - b^2) = 0,$$

which factors as

$$(\lambda - a - b)(\lambda - a + b) = 0.$$

Hence, the eigenvalues of \mathbf{A} are $\lambda_1 = a + b$ and $\lambda_2 = a - b$.

2.4 Eigenvalues of Epstein vectors

For $\lambda_1 = a + b$, we solve

$$(\mathbf{A} - \lambda_1 I)\mathbf{v} = 0, \quad \begin{pmatrix} a - \lambda_1 & b \\ c & d - \lambda_1 \end{pmatrix} \mathbf{v} = 0.$$

This system yields $v_1 = -v_2$, so the first Epstein vector is

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

For $\lambda_2 = a - b$, we solve

$$(\mathbf{A} - \lambda_2 I)\mathbf{v} = 0, \quad \begin{pmatrix} a - \lambda_2 & b \\ c & d - \lambda_2 \end{pmatrix} \mathbf{v} = 0.$$

This system yields $v_1 = v_2$, so the second Epstein vector is

$$\mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

3 Real-World Example: A Simple System

Consider a system consisting of two masses. Let the displacement from equilibrium be denoted by $x_1(t)$ and $x_2(t)$. Under standard assumptions (Hooke's law), the equations of motion can be written in matrix form as

$$\frac{d^2}{dt^2} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = -\mathbf{K} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},$$

where

$$\mathbf{K} = \begin{pmatrix} k_1 & k_2 \\ k_3 & k_4 \end{pmatrix}.$$

To analyze the stability of the system, we seek solutions of the form

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \mathbf{v} e^{i\omega t},$$

where \mathbf{v} is a constant vector and ω is a scalar. Substituting this into the equations yields

$$-\omega^2 \mathbf{v} = -\mathbf{K} \mathbf{v},$$

or equivalently,

$$\mathbf{K} \mathbf{v} = \omega^2 \mathbf{v}.$$

Thus, the eigenvalues ω^2 are the eigenvalues of the matrix \mathbf{K} , and the corresponding eigenvectors \mathbf{v} describe the modes of vibration.

Computing the eigenvalues of \mathbf{K} gives $\omega_1^2 = k_1 + k_2$ and $\omega_2^2 = k_3 + k_4$. Therefore, the system has two normal modes. The associated eigenvectors indicate that in the first mode, the masses move in phase, while in the second mode they move out of phase.

3.1 Interpretation

In this context, eigenvalues represent the squared angular frequencies of the system's normal modes, which are physically observable quantities. The eigenvectors represent the relative displacements of the masses in each mode. The transformation defined by the eigenvectors decouples the system's motion into independent harmonic oscillators.

4 Implications

The analysis of the eigenvalues and eigenvectors of the stiffness matrix \mathbf{K} provides a clear understanding of the system's dynamic behavior. It identifies the natural frequencies and the corresponding patterns of motion for the masses. This information is crucial for designing systems that avoid resonance and for understanding the underlying physics of the mechanical system. The decoupling of the equations of motion into independent harmonic oscillators simplifies the analysis and provides a powerful tool for predicting the system's response to external forces.

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4.1 Conclusion

The Epstein vectors
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